

## Übungsblatt zur Rechenfertigkeit

$$\begin{aligned} 1a) \quad & 5ab^2 - b^2 + a^3 + b^2 - 9ab^2 \\ & = -4ab^2 + a^3 \end{aligned}$$

$$\begin{aligned} b) \quad & 5b \cdot (-2a) + 2a \cdot (5b) \\ & = -10ab + 10ab \\ & = 0 \end{aligned}$$

$$\begin{aligned} c) \quad & -9ab^2 - 4a^2b + 2a(5b^2) \\ & = -9ab^2 - 4a^2b + 10ab^2 \\ & = ab^2 - 4a^2b \end{aligned}$$

$$\begin{aligned} d) \quad & (2w-4)(5w-4) \\ & = 10w^2 - 8w - 20w + 16 = 10w^2 - 28w + 16 \end{aligned}$$

$$\begin{aligned} e) \quad & (x+y)(z-a) \\ & = xz - xa + yz - ya \end{aligned}$$

$$\begin{aligned} 2a) \quad & 3y^3 + 2y^2 + y \\ & = y(3y^2 + 2y + 1) \end{aligned}$$

$$\begin{aligned} b) \quad & 2m^2 - 4m^3 + 6m^4 \\ & = 2m^2(1 - 2m + 3m^2) \end{aligned}$$

$$\begin{aligned} c) \quad & 21xy^2 - 12x^2y + 6xy \\ & = 3xy(7y - 4x + 2) \end{aligned}$$

$$3a) (2x+2)^2 = 4x^2 + 8x + 4$$

$$b) (6u-2)^2 = 36u^2 - 24u + 4$$

$$c) (3t-9w) \cdot (3t+9w) = 9t^2 - 81w^2$$

$$d) (5x+2y) \cdot (5x-3y)$$

$$= 25x^2 - 15xy + 10xy - 6y^2$$

$$= 25x^2 - 5xy - 6y^2$$

$$e) (\sqrt{2} + \sqrt{8})^2 = 2 + 2\sqrt{16} + 8$$

$$= 2 + 8 + 8 = 18$$

$$f) (\sqrt{3x} + \sqrt{27x})^2$$

$$= 3x + 2\sqrt{3 \cdot 27 \cdot x^2} + 27x$$

$$= 3x + 2 \cdot 9x + 27x$$

$$= 48x$$

$$4a) (2^2)^5 \cdot 2^3 = 2^{10} \cdot 8 = 1024 \cdot 8$$

$$= 8192$$

$$b) \frac{8^4 : 4^4}{2^3} = \frac{\left(\frac{8}{4}\right)^4}{2^3} = \frac{2^4}{2^3} = 2$$

$$c) (9^{-3})^{-2} : 3^6 = \frac{9^6}{3^6} = \left(\frac{9}{3}\right)^6 = 3^6 = 729$$

$$d) \frac{(16^{-2})^{-4} : 8^8}{2^6} = \frac{\left(\frac{16}{8}\right)^8}{2^6} = \frac{2^8}{2^6} = 2^2 = 4$$

$$5a) x^{-5} = \frac{1}{x^5}$$

$$b) (3x^2)^{-3} = \frac{1}{3^3 x^6} = \frac{1}{27x^6}$$

$$c) x^{\frac{1}{9}} = \sqrt[9]{x}$$

$$d) (2x)^{\frac{2}{5}} = \sqrt[5]{(2x)^2}$$

$$e) x^{-\frac{1}{11}} = \frac{1}{\sqrt[11]{x}}$$

$$f) (x^2)^{-\frac{3}{5}} = x^{-\frac{6}{5}} = \frac{1}{x \sqrt[5]{x}}$$

$$6a) \sqrt{16} - \sqrt{9} = 4 - 3 = 1$$

$$b) \sqrt{16+9} = \sqrt{25} = 5$$

$$c) \sqrt{16 \cdot 9} = 4 \cdot 3 = 12$$

$$d) \frac{\sqrt{16}}{\sqrt{9}} = \frac{4}{3}$$

$$7a) \frac{4}{\sqrt{11}} = \frac{4\sqrt{11}}{11}$$

$$b) \frac{1}{\sqrt{a}} = \frac{\sqrt{a}}{a}$$

$$c) \frac{5}{2\sqrt{5}} = \frac{\sqrt{5}}{2}$$

$$d) \frac{1}{2-\sqrt{3}} = \frac{2+\sqrt{3}}{2-3} = -2+\sqrt{3}$$

$$8a) -\frac{7}{2}x - 3 = -3x + 12 \quad | +3 \quad | -3x$$

$$-\frac{7}{2}x - \frac{6}{2}x = 15$$

$$-\frac{13}{2}x = 15$$

$$x = -\frac{15 \cdot 2}{13} = -\frac{30}{13}$$

$$b) \frac{7}{4} \cdot x - 2 = x + 28 \quad | -x + 2$$

$$\frac{7}{4}x - x = 30$$

$$\frac{3}{4}x = 30$$

$$x = 40$$

$$c) 0,2x^2 = -0,5x - 0,2$$

$$0,2x^2 + 0,5x + 0,2 = 0$$

$$x_{1/2} = \frac{-0,5 \pm \sqrt{0,25 - 4 \cdot 0,2 \cdot 0,2}}{2 \cdot 0,2}$$

$$= \frac{-0,5 \pm \sqrt{0,25 - 0,16}}{0,4}$$

$$= \frac{-0,5 \pm 0,3}{0,4} \quad x_1 = -\frac{1}{2}$$

$$x_2 = -2$$

$$d) -\frac{1}{3}x + x^2 = \frac{2}{9}$$

$$x^2 - \frac{1}{3}x - \frac{2}{9} = 0$$

$$x_{1/2} = \frac{\frac{1}{3} \pm \sqrt{\frac{1}{9} - 4 \cdot (-\frac{2}{9})}}{2}$$

$$= \frac{\frac{1}{3} \pm 1}{2}$$

$$x_1 = \frac{2}{3}$$

$$x_2 = -\frac{1}{3}$$

$$8e) -2 = \frac{+ - 3x}{x}$$

$$-2x = 7 - 3x \quad | + 3x$$

$$x = 7$$

$$f) 1 + \frac{3}{x} = \frac{10}{x^2} \quad | \cdot x^2$$

$$x^2 + 3x = 10$$

$$x^2 + 3x - 10 = 0$$

$$x_{1/2} = \frac{-3 \pm \sqrt{3 + 4 \cdot 10}}{2}$$

$$= \frac{-3 \pm 7}{2}$$

$$x_1 = 2$$

$$x_2 = -5$$

$$9a) \frac{3x}{x^2 - x} = \frac{3}{(x-1)}$$

$$\mathbb{D} = \mathbb{R} \setminus \{0, 1\}$$

$$b) \frac{x^2 - 8x + 16}{2x^2 - 8x} = \frac{(x-4)^2}{2x(x-4)} =$$
$$= \frac{(x-4)}{2x}$$

$$\mathbb{D} = \mathbb{R} \setminus \{0, 4\}$$

$$c) \frac{18xy^{-12}y}{9x^2-4} = \frac{18xy^{-11}}{9x^2-4}$$

$$\mathbb{D} = \mathbb{R} \setminus \left\{ \frac{2}{3} \right\}$$

$$10a) \text{ I) } 2x - 3y = -4$$

$$\text{II) } -x + y = 1 \Rightarrow *y = 1 + x$$

$$\text{in I) } 2x - 3(1+x) = -4$$

$$2x - 3 - 3x = -4$$

$$-x = -1$$

$$x = 1$$

$$y = 1 + 1 = 2$$

$$b) \text{ I) } 4x - 2y = -6$$

$$\text{II) } 5x + 3y = -2$$

$$\text{I) } 2x - y = -3$$

$$\Rightarrow y = 3 + 2x$$

$$\text{in II) } 5x + 3(3 + 2x) = -2$$

$$5x + 9 + 6x = -2$$

$$11x = -11$$

$$x = -1$$

$$y = 3 + 2(-1) = 1$$