

Lösung Lambacher-Schweizer S. 136 Nr. 12

$$f(x) = x^3 - 2x^2 + 1$$

Streckung um  $\frac{1}{2}$  in x- Richtung: Faktor 2 vor dem x:

- i)  $f_i(x) = f(2x)$   
 $= (2x)^3 - (2x)^2 + 1$   
 $= 8x^3 - 8x^2 + 1$
- ii)  $f_{ii}(x) = f_i(-x)$   
 $f_{ii}(x) = 8(-x)^3 - 8(-x)^2 + 1$   
 $= -8x^3 - 8x^2 + 1$
- iii)  $f_{iii}(x) = -f_{ii}(x) - 3$   
 $= -(-8x^3 - 8x^2 + 1) - 3$   
 $= 8x^3 + 8x^2 - 4$
- iv)  $f_{iv}(x) = \frac{1}{8} \cdot f_{iii}(x)$   
 $= \frac{1}{8} \cdot (8x^3 + 8x^2 - 4)$   
 $= x^3 + x^2 - \frac{1}{2}$
- v)  $f_v(x) = f_{iv}(2x)$   
 $= (2x)^3 + (2x)^2 - \frac{1}{2}$   
 $= 8x^3 + 4x^2 - \frac{1}{2}$
- vi)  $f_{vi}(x) = f_v(x + 1) + \frac{1}{2}$   
 $= 8(x + 1)^3 + 4(x + 1)^2 - \frac{1}{2} + \frac{1}{2}$   
 $= 8(x^3 + 3x^2 + 3x + 1) + 4(x^2 + 2x + 1) + 0$   
 $= 8x^3 + 24x^2 + 24x + 8 + 4x^2 + 8x + 4$   
 $= 8x^3 + 28x^2 + 32x + 12$
- vii)  $f_{vii}(x) = \frac{1}{4} \cdot f_{vi}(x)$   
 $= \frac{1}{4} \cdot (8x^3 + 28x^2 + 32x + 12)$   
 $= 2x^3 + 7x^2 + 8x + 3$
- viii)  $f_{viii}(x) = f_{vii}(x - 1) - 1$   
 $= 2(x - 1)^3 + 7(x - 1)^2 + 8(x - 1) + 3 - 1$   
 $= 2(x^3 - 3x^2 + 3x - 1) + 7(x^2 - 2x + 1) + 8x - 8 + 2$   
 $= 2x^3 - 6x^2 + 6x - 2 + 7x^2 - 14x + 7 + 8x - 6$   
 $= 2x^3 + x^2 - 1$
- ix)  $f_{ix}(x) = 4 \cdot f_{viii}\left(\frac{1}{2}x\right)$   
 $= 4 \cdot \left[ 2\left(\frac{1}{2}x\right)^3 + \left(\frac{1}{2}x\right)^2 - 1 \right]$   
 $= 4 \cdot \left[ 2 \cdot \frac{1}{8}x^3 + \frac{1}{4}x^2 - 1 \right]$   
 $= x^3 + x^2 - 4$